PREDICTION OF FATIGUE CRACK GROWTH IN PART-THROUGH CRACKED PIPES USING EXPONENTIAL MODEL AND GAMMA MODEL

A thesis submitted in partial fulfilment for award of the degree of

Master of Technology (Research)

by

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Dedicated

To

My Mother

National Institute of Technology
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This is to certify that the thesis entitled "**Prediction of fatigue crack growth in part-through cracked pipes using exponential model and gamma model**" submitted by **Mr. Pawan Kumar**, in partial fulfilment of the requirements for the award of **Master of Technology (Research)** in **Metallurgical and Materials Engineering**, at National Institute of Technology, Rourkela is an authentic work carried out by him under our supervision and guidance.

To the best of our knowledge, the matter presented in the thesis has not been submitted to any other University/Institute for the award of any Degree or Diploma.

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Prediction of fatigue crack growth in part-through cracked pipes using Exponential model and Gamma model

ABSTRACT

A pipe installation experiences fluctuating loading condition. It may also experience occasional high amplitude vibration (Seismic Vibration). The monitoring and modelling of fatigue crack growth becomes more significant if the pipes carry hazardous fluids. However there is no any model available for fatigue crack growth for pipes. The primary objective of the present investigation is to develop a fatigue crack growth model for part-through cracked pipes.

The pipes under investigation were of TP316L grade stainless steel. The four point bend test was conducted on pre-cracked specimen (notch angle 45° at the centre) using *Instron* 8800 machine. All the tests were conducted in air and at room temperature.

In the present investigation an attempt has been made to develop a fatigue life prediction methodology by using an Exponential Model and a Gamma Model for part-through cracked pipes. The predicted results are compared with experimental crack growth data. It has been observed that the results obtained from the two models are in good agreement with experimental data.

Keywords: fatigue crack propagation, part-through cracked pipes, exponential model, gamma model.

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NOMENCLATURE

a	semi circumferential crack length
a_i	initial crack length (mm), for exponential model
a_j	final crack length (mm), for exponential model
a_o	initial crack length (mm), for gamma model
a_1	final crack length (mm), for gamma model
A, B, C, D	curve fitting constants
da/dN	crack growth rate
K	stress intensity factor (MPa)
K_C	fracture toughness (MPa√m)
ΔK	stress intensity factor range (MPa√m)
m	specific growth rate
m_{ij}	specific growth rate in interval (i-j)
N	number of cycles
N^P_j	predicted fatigue life using exponential model
P	population
P_o	initial population
$P(t)$	population at any time

R	load ratio
R_i	Internal radius of specimen
t	thickness of specimen (exponential model)
w	thickness of specimen (gamma model)
σ_{bg}	bending stress

CHAPTER 1

INTRODUCTION

INTRODUCTION

1.1 Background

Pipe installations find wide applications in industry. They are used to carry fluids from one place to another. During their service they experience hoop stresses developed by the transporting fluid. As well as they are also exposed to fluctuating bending stresses along with the high amplitude seismic vibrations. These stress vibrations may initiate new fatigue cracks from a highly stressed region or promote extension of existing cracks. In several industries the pipe installations carry hazardous fluids [1, 2].Therefore, the monitoring of these installations in terms of initiation of crack and their subsequent growth are essential for stability of structure and safely. Several fracture mechanics based fatigue crack propagation models have been developed to predict fatigue crack propagation and life under different loading condition. In most of the cases life estimation are done by numerical integration. Mohanty *et. al.* in their recent work proposed an exponential model for prediction of fatigue crack growth for SENT specimen [3]. In the available literature CT and SENT geometry are only preferred for the crack growth modelling and life prediction. However, no such information is available for pipe geometry.

In present investigation attempts have been made to develop two different models to predict fatigue crack propagation in part-through cracked pipe under constant amplitude loading. The first involves modified exponential model. In the second model Gamma function is applied to predict fatigue crack growth behaviour.

In the present study the monitoring of crack was done with the help of a COD gauge. However, COD output and crack extension for pipe geometry if not available. Therefore, the first step of the experimentation involves the development of the crack growth calibration curve for the monitoring of the crack for part-through cracks. Usually the experimentally generated *a*-*N* data are scattered and cannot be directly utilised for the determination of fatigue crack growth rate (d*a*/d*N*).The smoothening of the *a*-*N* data is essential for the actual estimation of d*a*/d*N* and subsequently formulation of the models.

1.2 Objectives

The objective of present work is: To develop calibration curve, smoothening of *a*-*N* data and estimation of fatigue crack propagation life by using exponential model and gamma model.

1. To conduct fatigue crack propagation test of supplied stainless steel pipes under constant amplitude loading condition (R= 0.1).
2. Smoothening of *a*-*N* data obtained from experimentation.
3. To calibrate COD gauge and generation of fatigue crack profile for part-through cracked pipe.
4. To propose an exponential model to predict fatigue crack propagation in part-through cracked pipe under above mentioned loading condition.
5. To proposed a gamma model to predict fatigue crack propagation in part-through cracked pipe under constant amplitude loading condition.

1.3 Thesis structure

The concept of present investigation is presented through seven chapters. The first chapter presents an introduction of present work, 2nd chapter presents a brief review of literature. Chapter-3 describes the details of experimental procedure. Chaper-4 describes calibration of COD gauge and crack profile. Chapter-5 describes the formulation and validation of proposed exponential model under constant amplitude loading condition. Chapter-6 describes the formulation and validation of gamma model. Chapter-7 describes quantitative comparison of two proposed models with experimental data. Finally chapter-8 concludes remarks and discussion of possible future work.

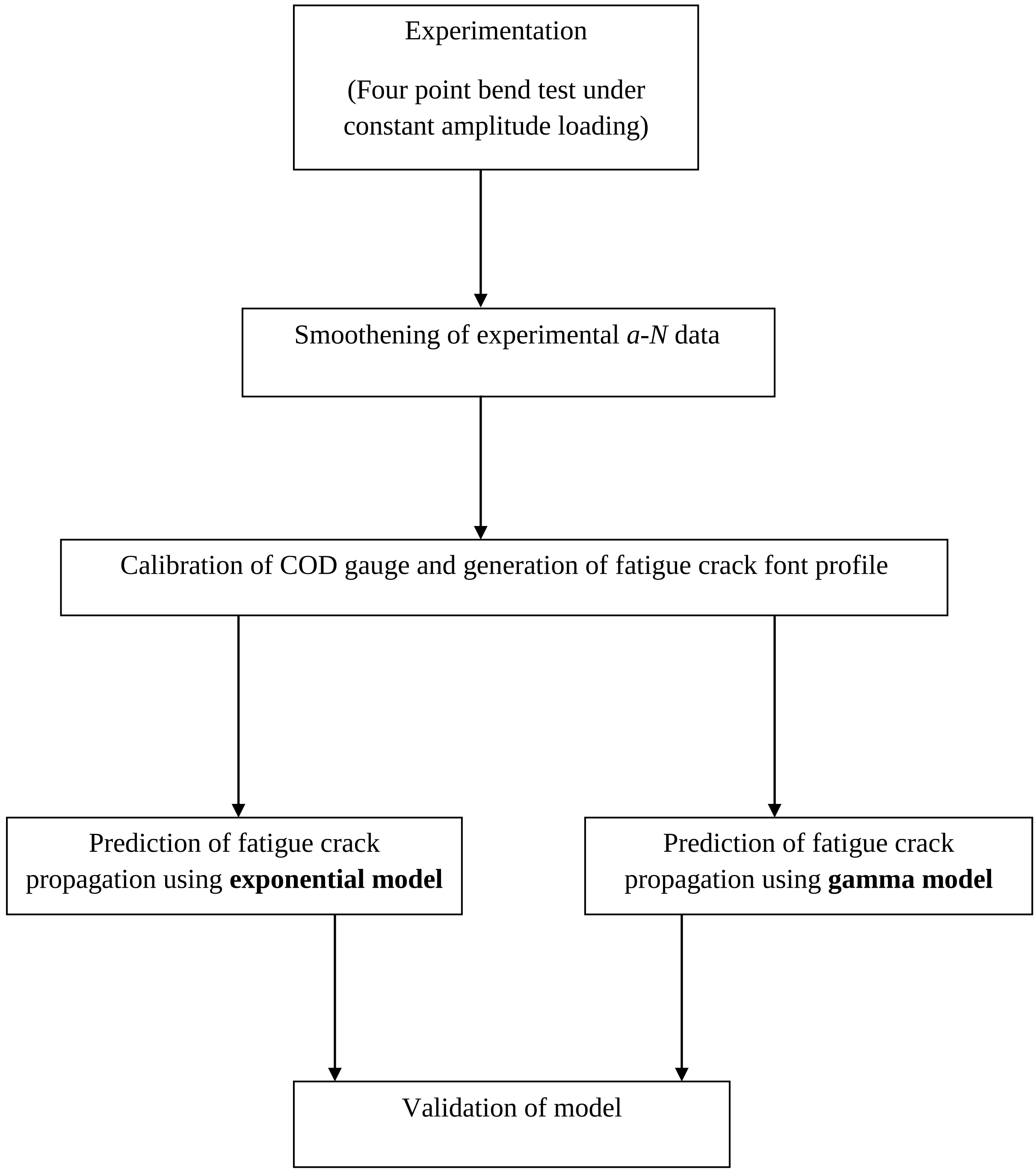

Fig. 1.1 Plan of work

CHAPTER 2

LITERATURE REVIEW

2.1 Introduction

Fatigue is defined as the process of progressive localized permanent structural damage occurring in a material subjected to conditions that produces fluctuating stresses at some point/s and that may culminate in cracks or complete fracture after certain number of fluctuations [4]. The stress value in case of fatigue failure is less than ultimate tensile stress and may be below yield stress limit of the material.

2.2 Fatigue Rate Curve

A typical fatigue rate growth curve, commonly referred as a d*a*/d*N* versus Δ*K* curve, is illustrated in Fig. 2.1 [4]. The curve is divided into Regions I, II and III on the basis of the slopes and nature of the curve.

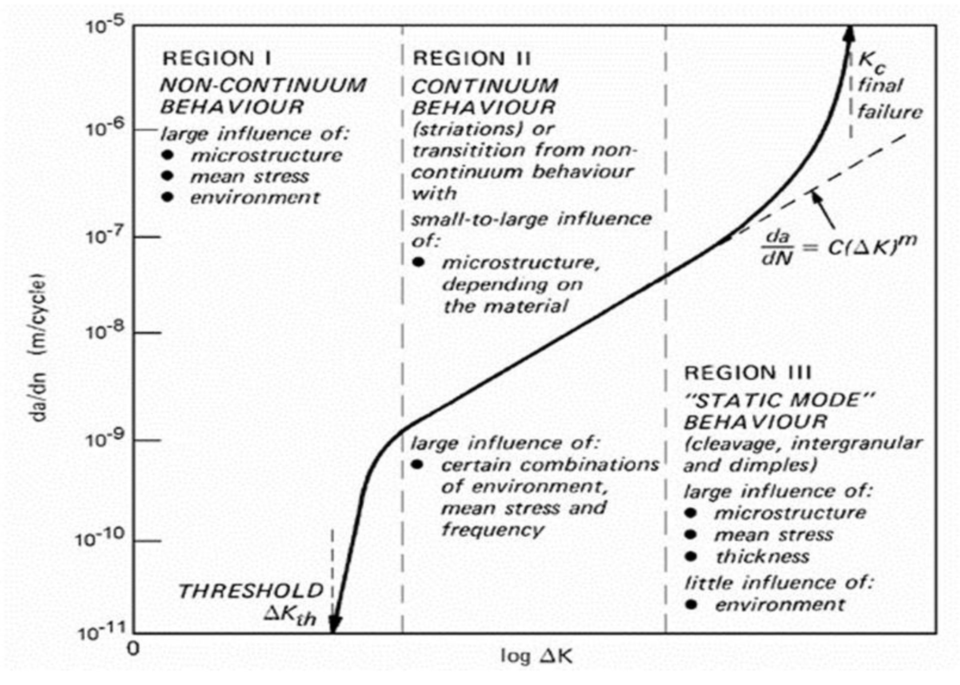

Fig. 2.1 Typical d*a*/d*N* versus log (Δ*K*) curve [4]

Region I

Region I represents the early development of a fatigue crack and the crack growth rate. This region is structural sensitive and largely influenced by the microstructural features of the material (such as grain size, phases present and their morphology etc.), the mean stress of the applied load cycle, the operating temperature and the environment. The most important feature of this region is the existence of a critical stress intensity factor range below which fatigue cracks do not propagate. This value of SIF is termed as threshold of stress intensity range, ΔK_{th} [4].

Region II

Region II represents the intermediate crack propagation range where the length of the plastic zone ahead of the crack tip is long compared with the mean grain size, but much smaller than the crack length [5]. The use of linear elastic fracture mechanics (LEFM) concepts is acceptable and the data follows a linear relationship between log da/dN and logΔK. This region corresponds to stable crack growth and the influence of microstructure, ductility, environment and thickness are small. The influence of the mean stress is probably the most significant [4].

Region III

Region III represents the fatigue crack growth at very high rates due to rapid and unstable crack growth just prior to final failure. The da/dN versus ΔK curve becomes steep and asymptotically approaches the fracture toughness K_c/ K_{Ic} of the material. The corresponding stress level is very high and causes a large plastic zone near the crack tip as compared to the specimen and crack geometry. Since crack extension in this region is associated with large yielding occurs, the influence of the nonlinear properties of the material cannot be ignored. The mean stress, materials microstructure and thickness have significant influence on the other hand the environment has little influence. Fatigue crack propagation analysis is very complex in this region but often ignored because it has little importance in most fatigue situations. The reason is the fatigue crack growth rates are very high and little fatigue life is involved [4].

2 .3 Fatigue crack propagation models

The study of fatigue crack propagation is always been a matter of research among scientists and researchers. For designing any structure, the fatigue crack behaviour has been an important parameter. Numerous attempts have been made in developing fatigue crack growth models for constant amplitude loading [4]. Most of the proposed models are based on integration of crack growth rate equation in order to estimate fatigue life. However, complicated numerical integration limits their applicability.

2.4 Constant amplitude loading models

The fatigue crack propagation analysis for constant amplitude loading condition is simplest as it is does not involve the interference of loading history and interaction. The available models vary in terms of factors and the number of curve fitting parameters. A few fatigue crack propagation models under constant amplitude loading condition are briefly discussed as follow:

Paris model

Paris *et al.* [6-9] proposed that the fatigue crack propagation follows a power law and describes as:

$$\frac{da}{dN} = C(\Delta K)^m$$

where C and n are material constants and ΔK is the stress intensity factor range given by $K_{max} - K_{min}$ .This law is simple and requires only two curve fitting constants to define fatigue crack propagation. However it valid for linear part (region II) of the curve and cannot be applied to other regions. It is also important to mention that there is no mention of stress ratio, an important parameter in the fatigue. The material properties are also ignored in this law.

Walker model

Walker *et al.* [10] included stress ratio (which has ignored by Paris) in fatigue crack propagation equation which is described as,

$$\frac{da}{dN} = C_b[(1-R)^{c_1} K_{max.}]^{n_b}$$

where R is the stress ratio under constant amplitude loading condition.

Forman model

The above cited laws are only applicable to the linear part of the crack extension region and unable to predict the behaviour at the stress intensity factor approaching the critical value. Forman suggested a new model, capable of describing the crack growth behaviour in region-III of the curve. The Forman law [11] is expressed as follows,

$$\frac{da}{dN} = \frac{C_F(\Delta K)^{m_y}}{(1-R)(K_C - K_{max})}$$

where K_c is the critical SIF corresponding to instability. The above relationship indicates that d*a*/d*N* tends to infinity as K_{max} approaches K_c.

McEvily model

McEvily proposed a model that relates the crack advance per cycle in the striation mode to the crack tip opening displacement and to include the threshold effect, which leads to following relationship [12],

$$\frac{da}{dN} = \frac{4A}{\pi\sigma_y E}(\Delta K^2 - \Delta K_{th}^2)$$

Frost and Pook model

Frost and Pook [13] proposed that the crack growth occurs under cyclic loading not as a consequence of any progressive structural damage but merely because unloading reshapes the crack tip at each cycle and this sequence is responsible for the formation of striations [14]. Therefore the increment of crack growth in each cycle can be related to the changing crack tip geometry during its opening and closing and a simple model based on,

$$\frac{da}{dN} = \frac{9}{\pi}\left(\frac{\Delta K}{E}\right)^2 \pi r^2 \qquad \text{for plane stress}$$

$$\frac{da}{dN} = \frac{7}{\pi}\left(\frac{\Delta K}{E}\right)^2 \qquad \text{for plane strain}$$

Zheng model

Lal and Weiss [15] proposed a static fracture model that indicates the occurrence of fracture when the maximum normal stress exceeded the critical fracture stress of the metal. Their model was moderately successful in predicting fatigue crack growth with some limitations.

Zheng [16] subsequently made changes by modifying the static fracture portion and introducing material constants related to the tensile properties of the metal. It is proposed that the crack opens elastically and the crack tip becomes blunt at the threshold value of SIF.

Wang model

Wang *et. al.* [17] proposed a damage accumulation theory, which considers the plastic component of the J integral as a damage factor resulting in a simple formula for the fatigue crack growth rate. The proposed damage accumulation theory assumes:

1. The total plastic strain energy density absorbed by the material is a constant prior to reaching its ultimate state.

2. The elastic strain energy density stored by the material does not cause damage and is released upon unloading.

Dowling and Begley Model

For situations of fatigue crack growth under large scale yielding conditions, where the stress intensity factor is no longer valid, Dowling and Begley [18-19] suggested to use the ΔJ integral as the fracture parameter.

The investigation of a pipe subject to bending moment with an equivalent plate subject to tension has been carried out by Mohammad Iranpour and Farid Taheri [20]. This study is done to avoid the complexity usually involved with experimental crack growth investigations of pipes with initial defect or surface flaw. This approach also minimizes the use of more complicated monitoring instruments, thereby reducing maintenance expenditure. This equivalency has been done for both finite element analysis and experimental investigations. A number of finite element analyses were carried out to verify both the values of the stress intensity factor (K) and the results of the interpolation function used in the computational simulation. Based on the FE simulations, crack front follows a semi-circular shape during its growth. Dimensionless relationship between stress intensity factor (K) of a pipe under bending moment and that of a plate under pure tension has been introduced. Numbers of experiments were performed to verify the validity of the proposed computational simulation. The analysis shows the acceptability of replacing a pipe subject to bending moment with an equivalent plate subject to tension.

Part-through cracks in pipes under cyclic bending is analysed by Andrea Carpinteri *et. al.* [21]. The crack propagation of a circumferential external surface crack in a metallic round pipe under cyclic bending load is studied through a two-parameter theoretical model. The finite element analysis is carried out to determine the stress-intensity factor distribution along the front of the surface crack, which is supposed to present an elliptical-arc shape. It is found that for each value of the relative crack depth the maximum stress-intensity factor is attained at the deepest point of surface flaw or defect front. An extended finite element analysis has been carried out to calculate the stress intensity factor along the defect front for other values of the geometrical parameters which define the part-through cracked configuration being considered.

Fatigue sensitive structures need crack growth models for their safe operations. Several models are available in literature; however, they mostly deal with SENT and CT specimen geometries. Though some attempts have been made to predict through-crack growth behaviour in pipes, the same has not been attempted for part-through cracked pipes, a condition essential for the safe operation of the pipe installations.

CHAPTER 3

EXPERIMENTAL INVESTIGATION

Experimental Investigations

3.1 Introduction

The fatigue crack growth tests were conducted on TP316L grade of stainless steel pipes. All the tests were conducted in a servo-hydraulic dynamic testing machine (*Instron* 8800) using part-through cracked pipe specimen under load control mode. A four point bend fixture was fabricated for conducting fatigue crack growth tests. Before conducting the test, COD gauge was calibrated for part-through cracked pipe specimens at notch angle 45°. All the tests were conducted in air and at room temperature.

3.2 Test specimen and accessories

The chemical composition of the material is presented in Table 3.1.

Table 3.1 Chemical Composition of TP316L stainless steel

Element	Weight (%)
Carbon	0.03
Manganese	2.00
Silicon	0.75
Chromium	16-18
Nickel	10-14
Phosphorus	0.045
Sulphur	0.030
Nitrogen	0.10
Molybdenum	2-3
Iron	balance

The Tensile properties of flat specimens fabricated from straight pipe were determined as per ASTM E8 standards and is given in Table 3.2. The LLD (load-load line displacement) diagram is shown in fig. 3.1.

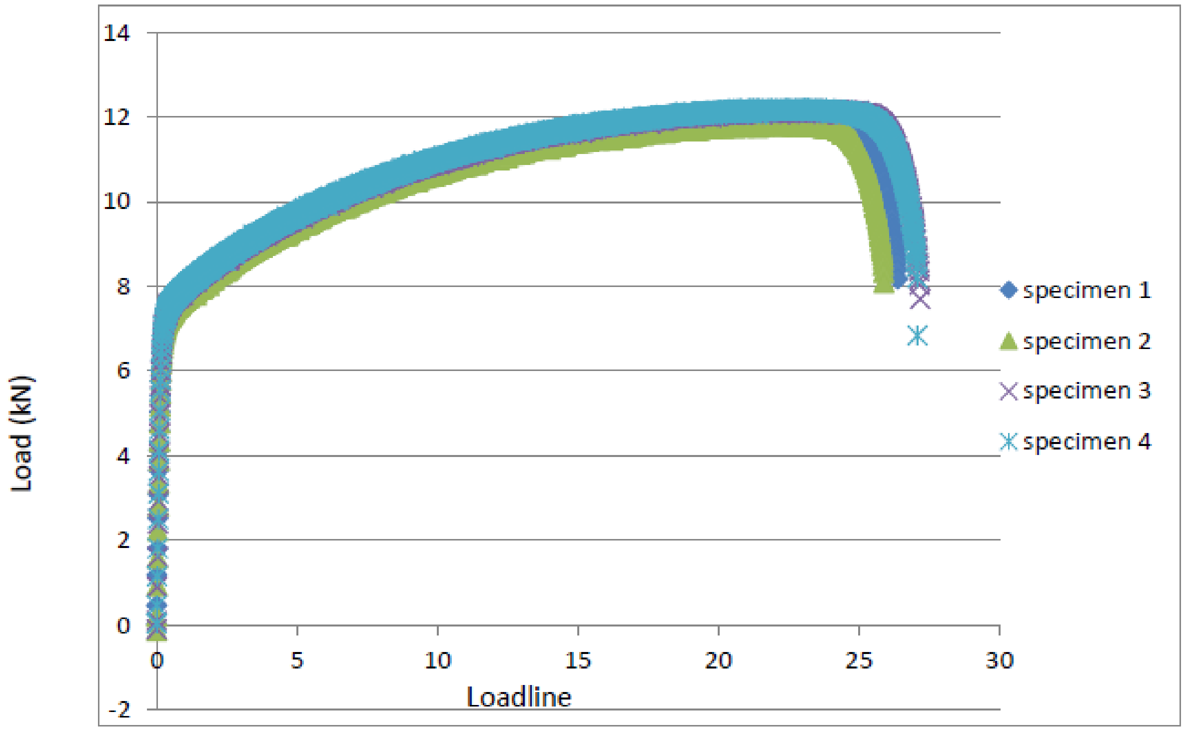

Fig. 3.1 Experimental results of Load vs. Load line displacement

Table 3.2 Mechanical Properties of TP316L stainless steel

Young's modulus (E)	220GPa
Poisson's ratio(μ)	0.3
Yield strength	366MPa
Ultimate tensile strength	611MPa

3.3 Specimen geometry

The pipes were of 60 mm outer diameter having 9 mm wall thickness. The pipe specimens had a notch at the outer wall in the circumferential direction. Straight surface notches of angle ($2\theta=45^o$) from the centre of pipe were made on the outer circumference by wire EDM which is shown on Figs. 3.2 and 3.3. The detailed dimensions of the specimen and notch are given in Table 3.3. The full length pipe specimen is shown in Fig. 3.4. Fig. 3.5 shows the cross-section after fracture.

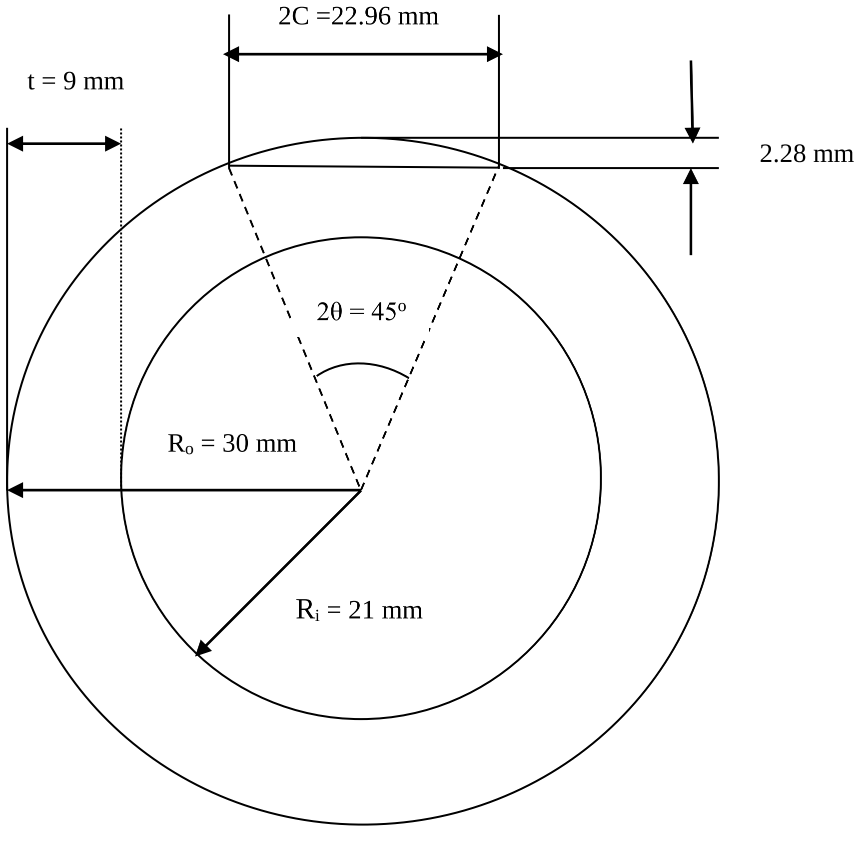

Fig. 3.2 Cross section of specimen

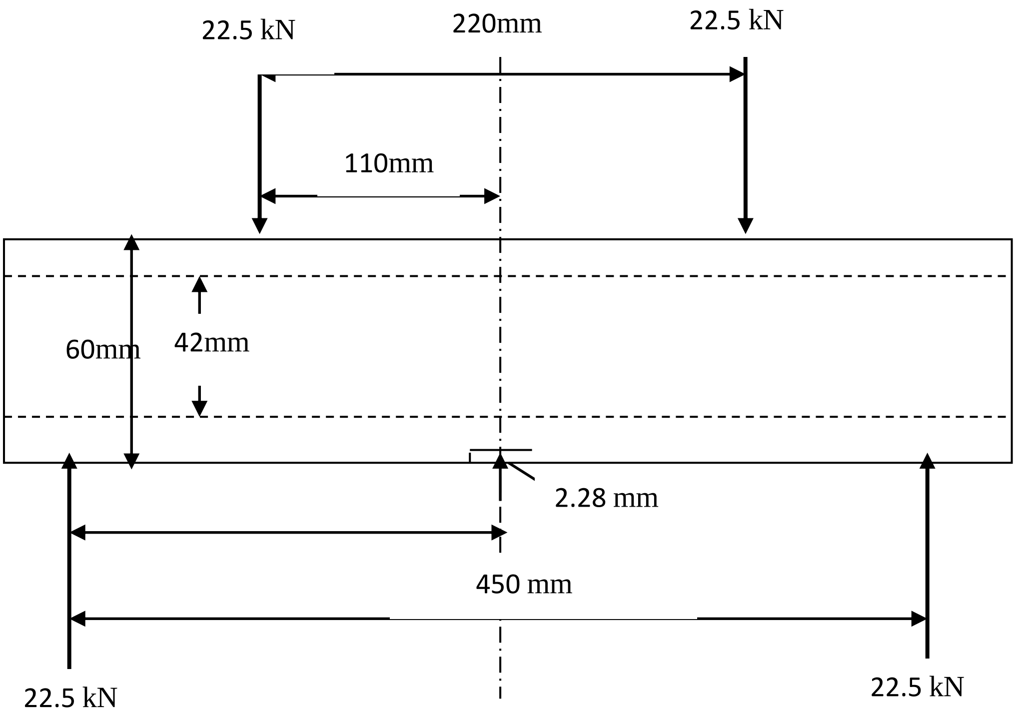

Fig. 3.3 Dimensions of pipe specimen with part-through notch at centre

Fig. 3.4 Test specimen

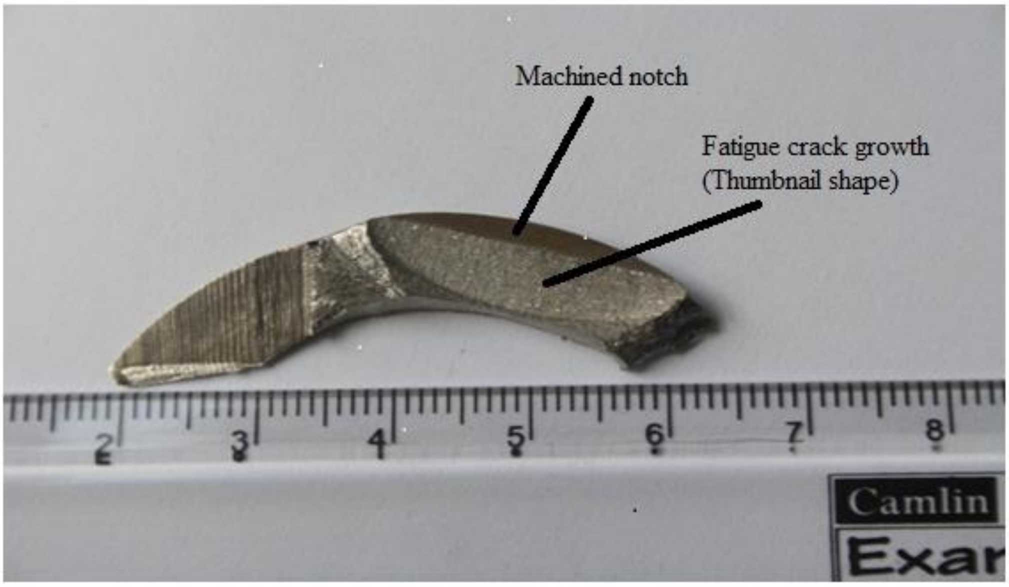

Fig. 3.5 Surface after fatigue

Table 3.3 Specimen and notch dimension of pipe

Specimen parameters	Dimension (mm)
Outer radius (R_0)	30
Inner radius (R_i)	21
Thickness (t)	9
Crack depth	2.28
Crack length (L)	23
Length of the specimen	505
Angle (2θ)	45^o

3.4 Test Equipment

The servo-hydraulic dynamic testing machine (*Instron* 8800) having load cell of capacity 250 KN were used for fatigue test which is interfaced to a computer for machine control and data acquisition. Fig 3.6 shows a photograph of the test set-up along with the test specimen.

Fig. 3.6 *Instron* 8800 dynamic testing machine

3.5 Test condition

The tests were conducted at room temperature and laboratory air environment under constant amplitude sinusoidal loading at stress ratio of 0.1 and frequency 4 Hz. The load range applied during the fatigue crack initiation and growth test was of the order of 40.5 KN.

CHAPTER 4

CALIBRATION OF COD GAUGE AND GENERATION OF FATIGUE CRACK PROFILE

4.1 Introduction

In present work calibration of COD gauge was done using multiple specimen technique. The crack profile was measured and plotted with the help of optical travelling microscope.

4.2 Calibration of COD gauge

Pipes with straight notches were used for calibration of COD gauge. The COD calibration curve (Del. COD vs. measured crack length along the pipe thickness) is shown in Fig. 4.1.

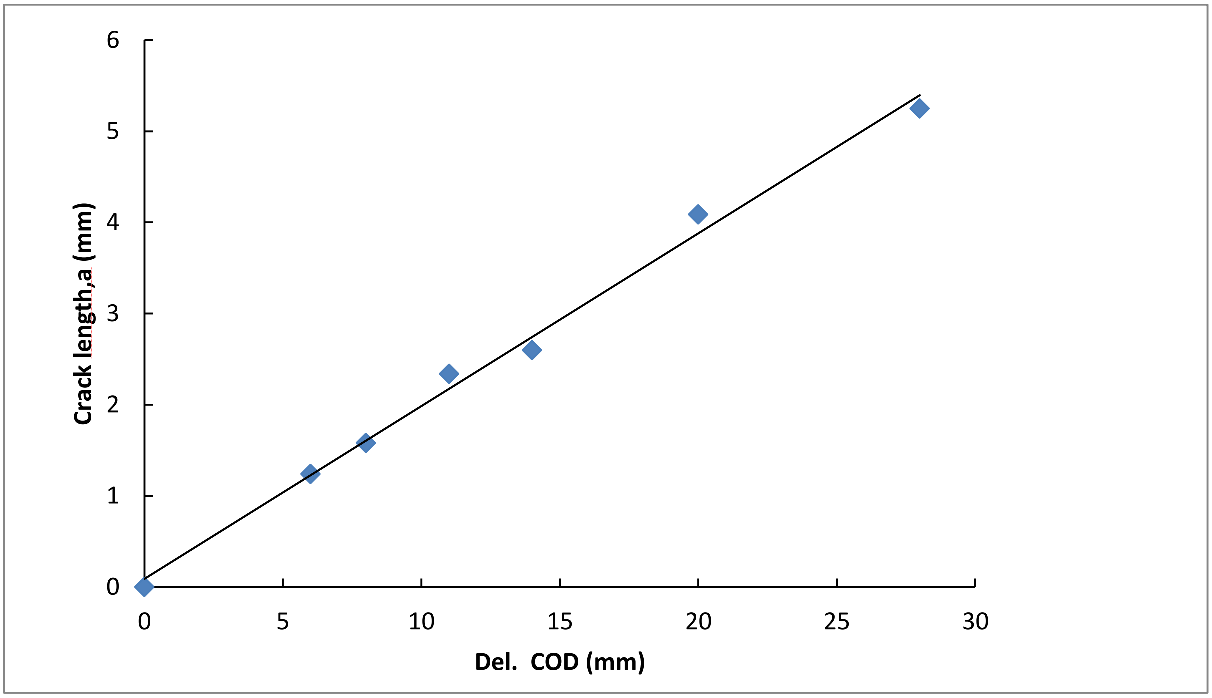

Fig. 4.1 Calibration of COD gauge

The crack profiles obtained by using multiple specimens are shown in Fig. 4.2. The specimens were dipped in liquid nitrogen and then broken by hammering. Depth of cracks in the thickness direction was then measured with the help of a travelling microscope. From the crack profile it is seen that at the early stage of crack extension the crack propagates along thickness direction, and subsequently the crack propagates in circumferential direction also.

It is found that the crack front profile is semi-elliptical in nature for smaller crack depth, but the crack front shape is flattened as the crack depth increases. Propagation in circumferential direction reduces the SIF at the main crack front. This may be the probable cause of flattening of the main crack front after the crack has grown some distance in the through-thickness direction [22].

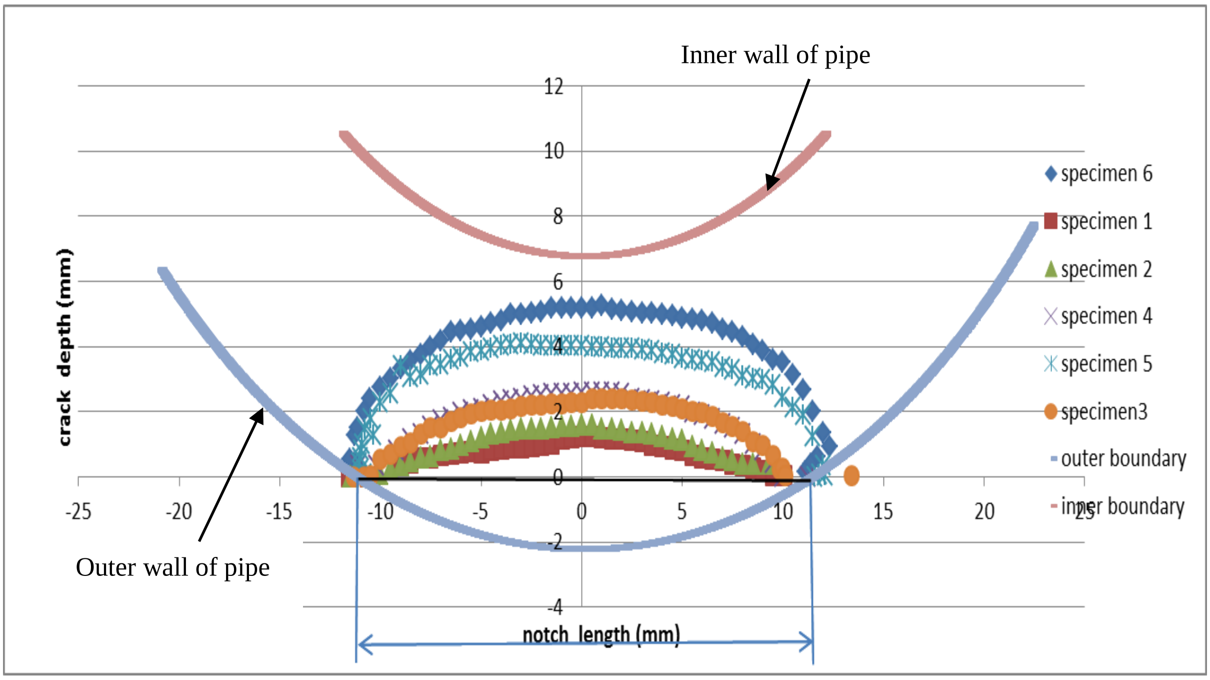

Fig. 4.2 Crack profile [22]

CHAPTER 5

PREDICTION OF FATIGUE CRACK PROPAGATION USING EXPONENTIAL MODEL

Fatigue crack propagation, a natural physical process of material damage, is characterised by rate of increase of crack length (*a*) with number of cycles (*N*). It requires a discrete set of crack length vs. Number of cycle data generated experimentally. Unlike monotonic tests, fatigue test data are usually scattered. Therefore curve fitting of experimental *a*-*N* data was done which are usually scattered (Fig. 5.1).

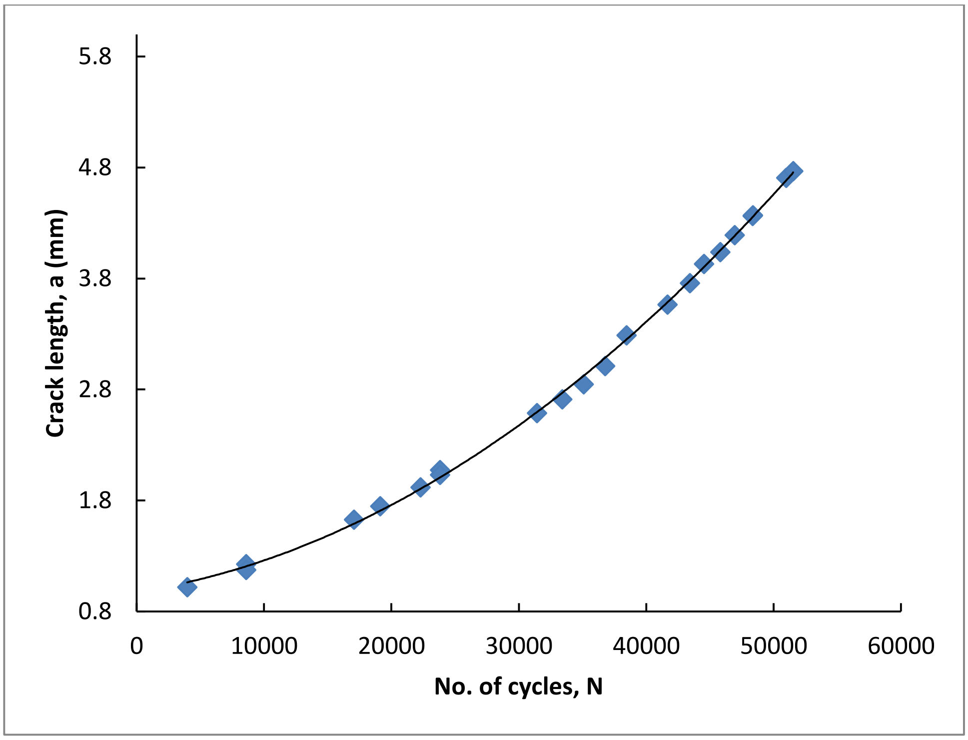

Fig. 5.1 Curve fitting of experimental data (crack length Vs number of cycles)

5.1 Introduction:

The exponential model was developed by Mohanty *et. al.* [23-25] for prediction of fatigue crack growth in SENT specimen for constant amplitude loading as well as variable amplitude loading. However, this model was not developed for pipe specimens. In the present investigation an attempt has been made to use the exponential model for fatigue crack propagation in part-through cracked pipes subjected to constant amplitude loading.

This model is based on the exponential nature of fatigue crack propagation with number of loading cycles. The exponent (known as specific growth rate) of the proposed exponential model has been correlated with various physical variables like crack driving parameters, crack resisting parameter, and material properties in non-dimensional forms. Finally the validation of the model has been done with experimental data in order to compare its accuracy in predicting fatigue life in part-through cracked pipes.

5.2 Background and approach

Use of exponential model was first suggested by Thomas Robert Malthus (1766-1834) for the prediction of growth of human population/bacteria. He realized that any species could potentially increase in numbers according to an exponential series. The differential equation describing an exponential growth is

$$\frac{dP}{dt} = rP \qquad (5.1)$$

Where P is population, t is time and the quantity r in the above equation is the Malthusian parameter, also known as specific growth rate. The solution of the above differential equation is

$$P(t) = P_o\, e^{rt} \qquad (5.2)$$

This equation is called law of growth.

In our present research equation (5.2) is modified and form of exponential equation of the proposed exponential model is as follows:

$$a_j = a_i e^{m_{ij}\left(N_j - N_i\right)} \tag{5.3}$$

The exponent, i.e. specific growth rate (m_{ij}) is calculated by taking logarithm of the above equation as follows:

$$m_{ij} = \frac{ln\left(\frac{a_j}{a_i}\right)}{\left(N_j - N_i\right)}$$

In conventional differential equation model of Paris-Erdogan, there is a physical inconsistency when the constants of the crack growth rate equation are randomized as per dimensional analysis point of view [26]. In case of the proposed exponential model, this type of inconsistency does not arise as the specific growth rate m_{ij} is a dimensionless parameter. The specific growth rate m_{ij} is given by:

$$m = Al^3 + Bl^2 + Cl + D \tag{5.4}$$

where l is correlated with two crack driving forces ΔK and K_{max} as well as material parameters K_c, E, σ_{YS} and is represented by equation:

$$l = \left[\left(\frac{\Delta K}{K_C}\right)\left(\frac{K_{max}}{K_C}\right)\left(\frac{\sigma_{YS}}{E}\right)\right]^{1/4} \tag{5.5}$$

5.3 Formulation and validation of model:

The modified exponential equation is given as:

$$a_j = a_i e^{m_{ij}}(N_j - N_i) \qquad (5.6)$$

And,

$$m_{ij} = \frac{ln\left(\frac{a_j}{a_i}\right)}{(N_j - N_i)} \qquad (5.7)$$

Here N_j and N_i represent number of cycles in *j*th step and *i*th step respectively, and a_j and a_i are the crack lengths in *j*th step and *i*th step respectively. m_{ij} is specific growth rate in the interval (*i*-*j*).

The specific growth rate *m* is calculated for each step from experimental result of fatigue test (*a*-*N* data) according to equation (5.7).

The specific growth rate is correlated with another parameter *l* which takes into account two crack driving forces ΔK and K_{max} as well as material parameters K_c, E, σ_{YS} and is represented by equation:

$$l = \left[\left(\frac{\Delta K}{K_C}\right)\left(\frac{K_{max}}{K_C}\right)\left(\frac{\sigma_{YS}}{E}\right)\right]^{1/4} \qquad (5.8)$$

The different *m* and *l* values are fitted by a polynomial equation. The predicted *m* values are calculated for seven specimens by using formula:

$$m = Al^3 + Bl^2 + Cl + D \qquad (5.9)$$

where *A*, *B*, *C*, and *D* are curve fitting constants whose average value for seven specimens have been presented in the Table (5.1).

The stress intensity factor *K* has been calculated by equation [27]:

$$K = \sqrt{\pi a}\left(\sum_{i=1}^{3} \sigma_t\, f_t\, \left(\frac{a}{t}\,\frac{2c}{a}\,\frac{R_t}{t}\right) + \sigma_{bg}\, f_{bg}\, \left(\frac{a}{t}\,\frac{2c}{a}\,\frac{R_t}{t}\right)\right) \tag{5.10}$$

Here σ_{bg} is bending stress, and σ_t is axis-symmetrical stress which is zero in present case.

5.3.1 Validation of model

The predicted number of cycles or fatigue life is given by:

$$N_j^P = \frac{ln\left(\frac{a_j}{a_i}\right)}{m_{ij}} + N_i \tag{5.11}$$

The predicted values of specific growth rate (m_{ij}) of the tested specimen have been calculated by putting the average values of the curve fitting constants (for specimen nos. 1, 2, 3, 4, 5, 6, 7) in equation (5.9). For validation of proposed exponential model; fatigue life is calculated (for specimen no. 8) by using the equation (5.11)

5.4 Discussion

The basic aim of present work is to develop a fatigue crack propagation model for part-through cracked pipe without going through numerical integration process. The specific growth rate (*m*) is an important parameter of our model. The value of *m* is correlated with two crack driving forces (ΔK *and* K_{max}), and with material parameters fracture toughness (K_C), yield strength (σ_{YS}), Young's modulus (*E*) by curve fitting. The experimental *a*-*N* data of seven specimens were used for formulation of model, and its validation has checked for 8th specimen. The average value of curve fitting constants for pipes has been given in Table5.1. By using these constants fatigue life of a pipe specimen can be predicted. The predicted result by using exponential model has been compared with the experimental result. The *a*-*N* curve obtained from proposed exponential model and that obtained from experimental data have been compared [Fig. 5.2]. The *da/dN* – Δ*K* curves are also compared [Fig. 5.3]. It can be seen that the predicted results are in good with the experimental data. Also log (*da/dN*)-Δ*K* curve was plotted for TP316L stainless steel pipe (Fig. 5.4).

Table 5.1 Value of coefficients for exponential model

A	B	C	D
-359.484	+52.708	-2.578	0.0421

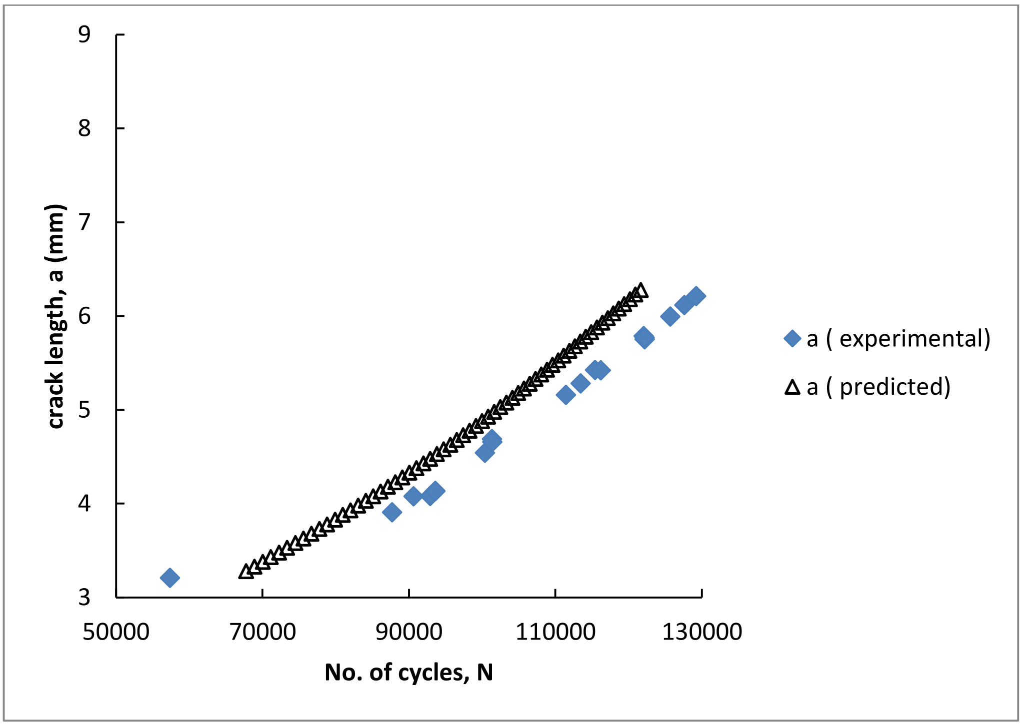

Fig. 5.2 *a*-*N* curve (exponential model)

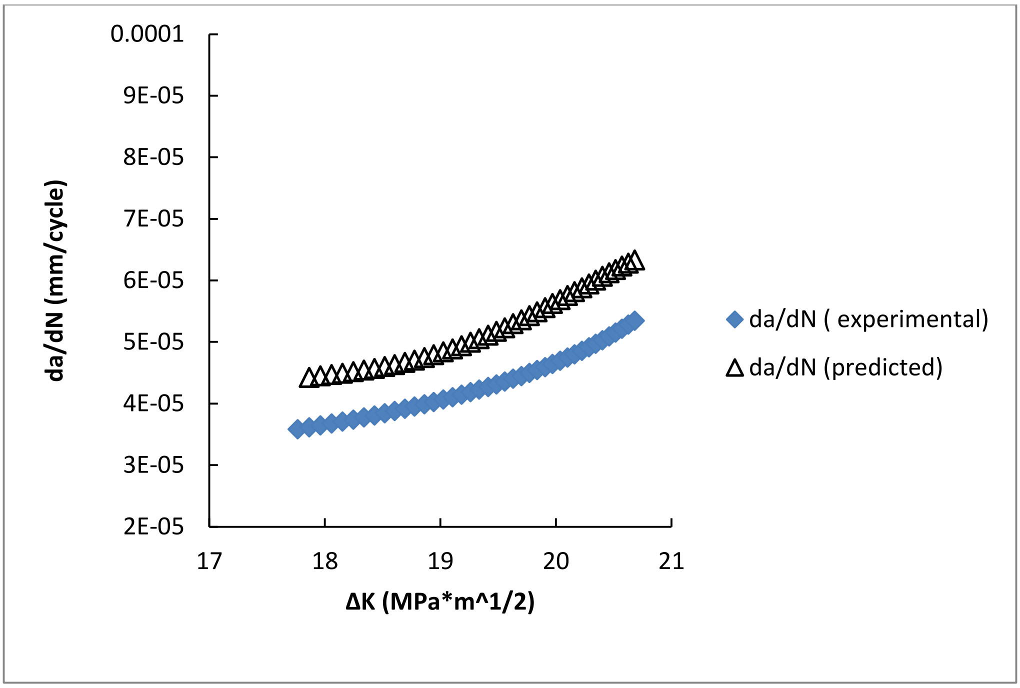

Fig. 5.3 d*a*/d*N*-Δ*K* curve (exponential model)

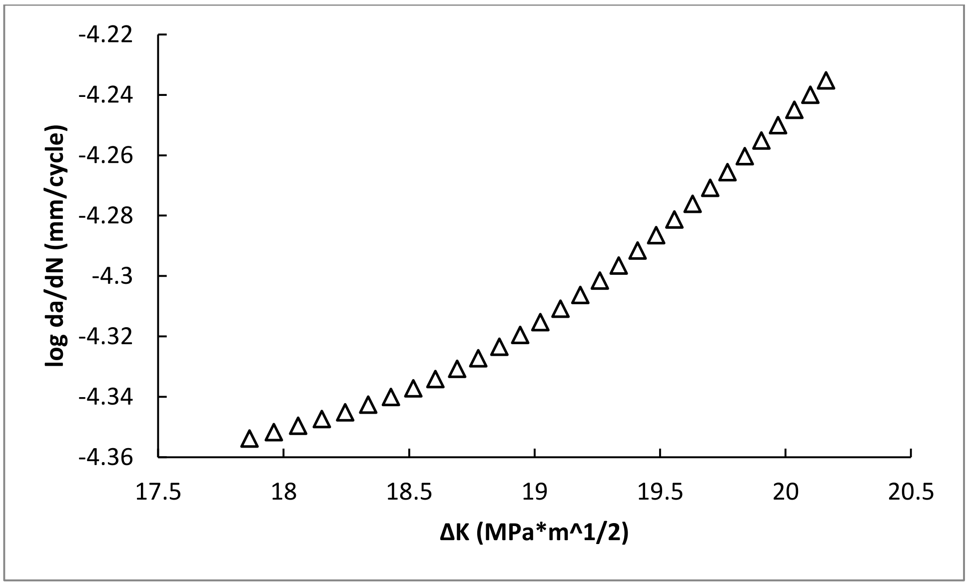

Fig. 5.4 log (d*a*/d*N*) –Δ*K* curve (exponential model)

CHAPTER 6

PREDICTION OF FATIGUE CRACK PROPAGATION USING GAMMA MODEL

PREDICTION OF FATIGUE CRACK PROPAGATION USING GAMMA MODEL

6.1 Introduction

Here a gamma model has been formulated to estimate the fatigue crack growth for part-through cracked pipe specimens. The main feature of the model is that the gamma function is correlated with various physical variables like crack driving parameters and materials parameters in non- dimensional form such that the proposed model can be used for different loading conditions. Finally the validation of model has been done with experimental data in order to compare its accuracy in predicting fatigue crack growth.

6.2 Background and approach

Gamma function is defined as the generalization of the factorial function to non-integral values, introduced by the Swiss mathematician Leonhard Euler in the 18th century. For a positive whole number *n*, the factorial (written as *n*!) is defined by $n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$. But this formula is meaningless if *n* is not an integer. To extend the factorial to any real number $n > 0$ (whether or not *n* is a whole number), the gamma function is defined as:

$$\Gamma(\mathrm{z}) = \int_0^N t^{z-1}\, e^{-t} dt, Re(z) > 0 \tag{6.1}$$

In our present investigation a modified gamma model has been proposed to predict crack growth in part-through cracked pipe. Here *t* is replaced by number of cycles *N*. The parameter *z* is chosen in such a way that it becomes non-dimensional and represents the properties that affect crack growth. The integral is chosen so that it is non-dimensional and represents crack growth at the end of a fixed number of loading cycles. Generally fatigue crack growth depends on the initial crack length, material properties and specimen geometry, loading conditions etc. The non-dimensional parameter is chosen in such a way so that it includes all these properties. Therefore formula for predicting the final crack length at the end of *N* cycle is given by:

$$\frac{ma_1}{w} = \int_0^N N^{\left(\frac{ma_0}{w}-1\right)} e^{-N} dN \qquad (6.2)$$

Here *z* has been replaced by [*m**(*a*/*w*)], where *w* is the specimen thickness, and *m* is defined as a non-dimensional parameter whose value remain approximately constant for a given cycle interval. The value of *m* includes all the properties which affect crack growth. Therefore value of *m* is given by:

$$m = Al^3 + \ Bl^2 + \ Cl + D \qquad (6.3)$$

where *A*, *B*, *C*, and *D* are curve fitting constants. The non-dimensional number *m* is correlated with another parameter *l* which takes into account two crack driving forces ΔK and K_{max} as well as material parameters K_c, *E*, σ_{YS} and is represented by equation:

$$l = \left[\left(\frac{\Delta K}{K_C}\right)\left(\frac{K_{max}}{K_C}\right)\left(\frac{\sigma_{YS}}{E}\right)\right]^{1/4} \qquad (6.4)$$

6.3 Formulation and validation of model

Fatigue crack growth behaviour depends upon initial crack length and load history. Therefore, while using gamma model each previous crack length is taken as initial crack length for the present step and non-dimensional number *m* is calculated for each step in incremental manner. The experimental *a*-*N* data have been used to determine the non-dimensional parameter *m* for each step using MATLAB programming.

6.3.1 Formulation of model

The modified gamma function is given as:

$$\frac{ma_1}{w} = \int_0^N N^{\left(\frac{ma_0}{w}-1\right)} e^{-N} dN \qquad (6.5)$$

Here *z* has been replaced by [m× (crack length/w)], where w is the specimen thickness, and m is defined as a non-dimensional parameter whose value remain approximately constant for a given cycle interval.

The value of *m* is approximately constant for a small cyclic interval. At first value of *m* has been assumed for a given interval of cycle. The values of a_0, w, a_1, N were given as input is made fixed for a particular interval of cycles. The value of *m* for which LHS and RHS of the equation (6.5) becomes nearly equal ($\pm$ 0.01) will be the value of m for given cyclic interval.

The RHS of equation (6.5) was solved with the help of MATLAB programming which is described as follow:

6.3.2 MATLAB Programming:

STEP 1: define the gamma function

C=(ma_o/w)-1

Put the respective values of m, a_o, w

Syms n;

f=n^(c)*exp(-n);

int(f,N1,N2)

STEP 2: copy the real part of gamma function and again put into MATLAB

STEP 3: compare LHS and RHS

If LHS-RHS< 0.01

The value of m has been accepted.

6.3.3 Validation of model:

The value of *m* is correlated with all the properties which affect crack growth. Therefore value of *m* is given by:

$$m = Al^3 + Bl^2 + Cl + D \qquad (6.6)$$

where *A*, *B*, *C*, and *D* are curve fitting constants whose average value for seven specimens have been presented in the table (6.1). The specific growth rate *m* is correlated with another parameter *l* which takes into account two crack driving forces ΔK and K_{max} as well as material parameters K_c, E, σ_{YS} and is represented by equation:

$$l = \left[\left(\frac{\Delta K}{K_C}\right)\left(\frac{K_{max}}{K_C}\right)\left(\frac{\sigma_{YS}}{E}\right)\right]^{1/4} \qquad (6.7)$$

The stress intensity *K* has been calculated by equation: [8]

$$K = \sqrt{\pi a}\left(\sum_{i=1}^{3} \sigma_t f_t \left(\frac{a}{t}\ \frac{2c}{a}\ \frac{R_t}{t}\right) + \sigma_{bg} f_{bg} \left(\frac{a}{t}\ \frac{2c}{a}\ \frac{R_t}{t}\right)\right) \qquad (6.8)$$

Here σ_{bg} is the bending stress and σ_t is the axis-symmetrical stress which is zero in present case.

The predicted crack length was calculated as:

$$a_1 = \frac{w}{m} * \int_0^N N^{\left(\frac{ma_0}{w}-1\right)} e^{-N} dN \qquad (6.9)$$

Here *m* has been given by equation (6.6) after putting average value of curve fitting constants (for seven specimens) and validate it by using eq. (6.9) for 8^{th} specimen.

6.4 Discussion

The experimental *a*-*N* data of specimen no. 1, 2, 3, 4, 5, 6, and 7 were used for formulation of model, and its validation has been checked by 8^{th} specimen. The predicted *a*-*N* curve obtained from proposed gamma model was compared with experimental test data (Fig. 6.1). The average values of curve fitting constants (for seven different specimens) for gamma model have been given in Table 6.1. The d*a*/d*N*-Δ*K* curves are illustrated in fig. 6.2 for the tested specimen for comparison. Also log (d*a*/d*N*)-Δ*K* curve was plotted for TP316L stainless steel pipe (Fig. 6.3).

Table 6.1 Value of coefficients for gamma model

A	B	C	D
23.33 E+07	35.12E+06	-17.65E+05	29.63E+03

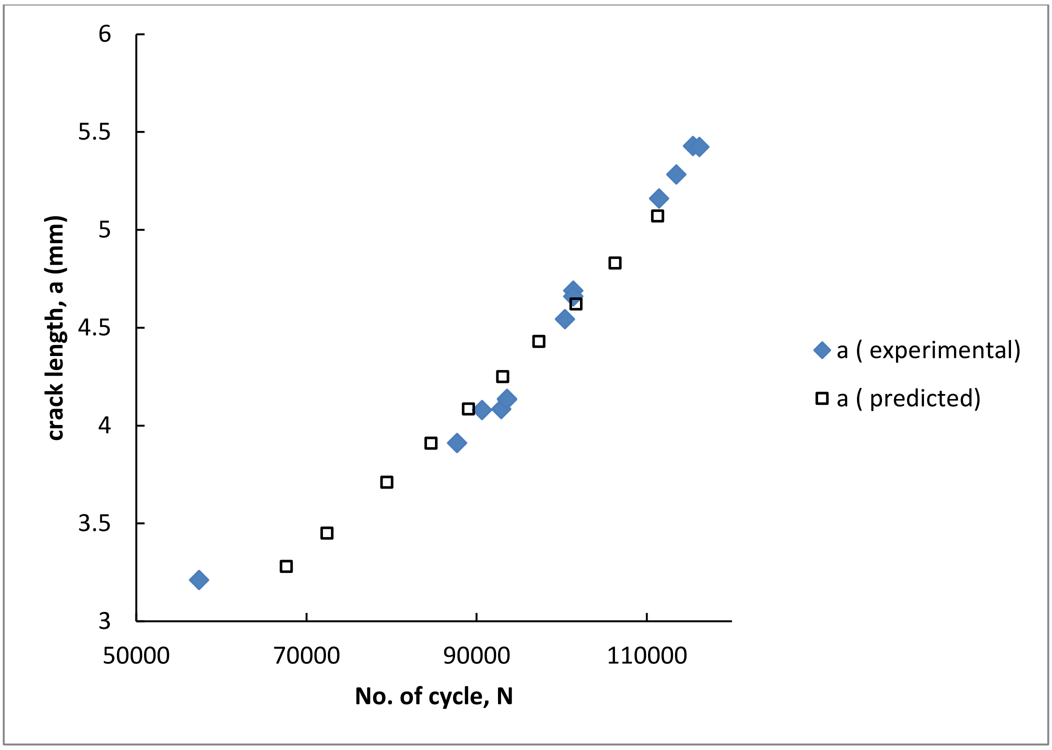

Fig. 6.1 *a-N* curve (gamma model)

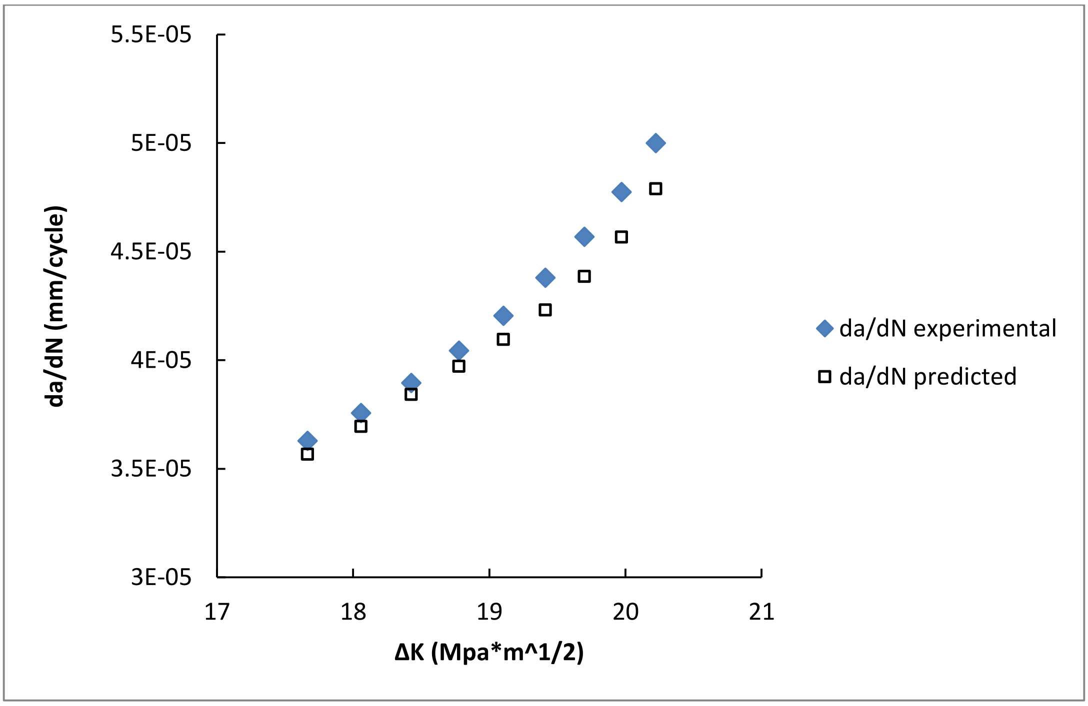

Fig. 6.2 d*a*/d*N*-Δ*K* curve (gamma model)

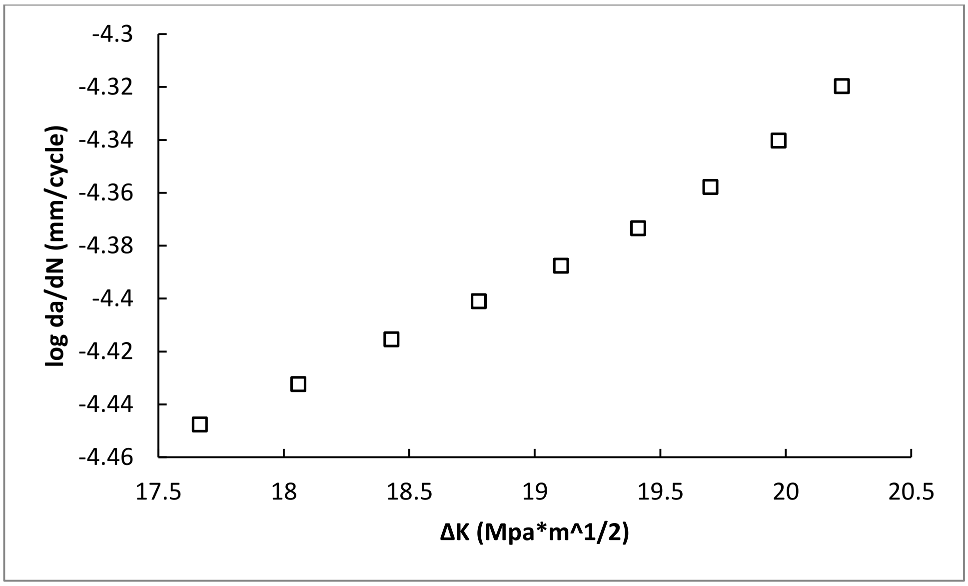

Fig 6.3 log (d*a*/d*N*) – Δ*K* curve (gamma model)

CHAPTER 7

RESULTS AND DISCUSSION

RESULTS AND DISCUSSION

7.1 Introduction

This chapter describes the performance characteristics of the proposed exponential model and the gamma model. There are three different evaluation criteria adopted to compare the prediction accuracy with experimental results.

7.2 Comparison of predicted and experimental results

The performance of exponential model and gamma model are evaluated by comparing the predicted results with experimental data for part-through cracked pipes under constant amplitude loading condition. There are three criteria that have been used for comparison of predicted results and experimental data, which are:

1. Percent deviation of predicted result from the experimental data as:

$$\%\,Deviation = \frac{Predicted\ result - Experimental\ result}{Experimental\ result} * 100$$

2. Prediction ratio which is defined as ratio between experimental data to predicted result as:

$$Prediction\ ratio\ P_r = \frac{Experimental\ data}{Predicted\ result}$$

3. Error bands i.e. the scatter of the predicted life in either side of experimental line within certain error limits.

Seven specimens have been used for prediction of fatigue crack propagation in part-through cracked specimen. The validity of proposed exponential model as well as gamma model has been checked for specimen no. 8.

The percentage deviations and the prediction ratio of exponential model as well as gamma model are presented in table 7.1 and 7.2. It is observed that percent deviation for exponential model is more than gamma model; however prediction ratio of both the models is approximately 1.0.

Table 7.1 Model Performances (for crack length)

Test specimen	% Dev (exponential model)	% Dev (gamma model)	Prediction ratio (exponential model)	Prediction ratio (gamma model)
TP316L stainless steel	5.80	1.74	0.94	0.99

Table 7.2 Model performances (for number of cycle)

Test specimen	% Dev (exponential model)	% Dev (gamma model)	Prediction ratio (exponential model)	Prediction ratio (gamma model)
TP316L stainless steel	3.66	1.02	1.038	1.01

The error band scattered evaluation for both exponential model and gamma model has been shown in Figs. (7.1-7.4). It has been observed that prediction by the proposed exponential model lie within 0.0% to -0.09% of experimental number of cycles and 0.0% to +0.06% of experimental crack length. However, the error band scatter of gamma model lies in the range of +0.03% to -0.03% for experimental number of cycles and +0.03 to -0.03% of experimental crack length.

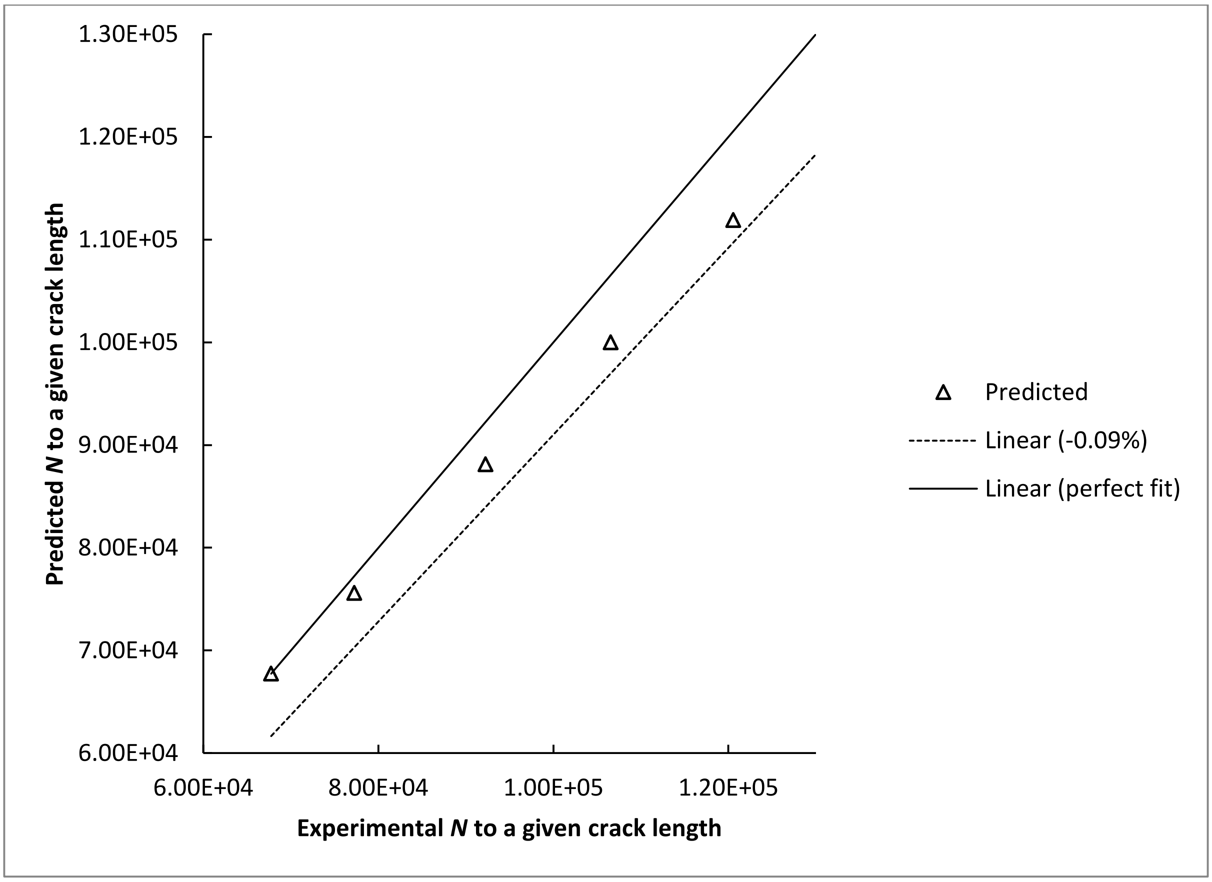

Fig. 7.1 Error band scatter for number of cycle (exponential model)

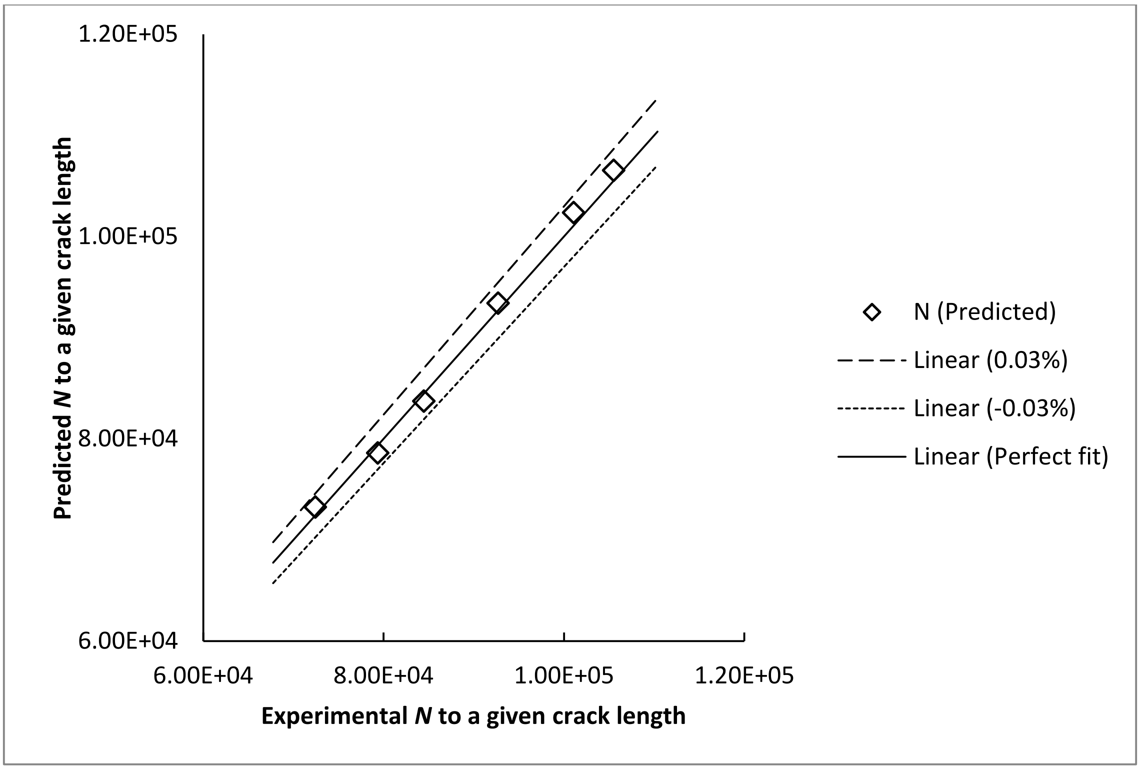

Fig. 7.2 Error band scatter for number of cycle (gamma model)

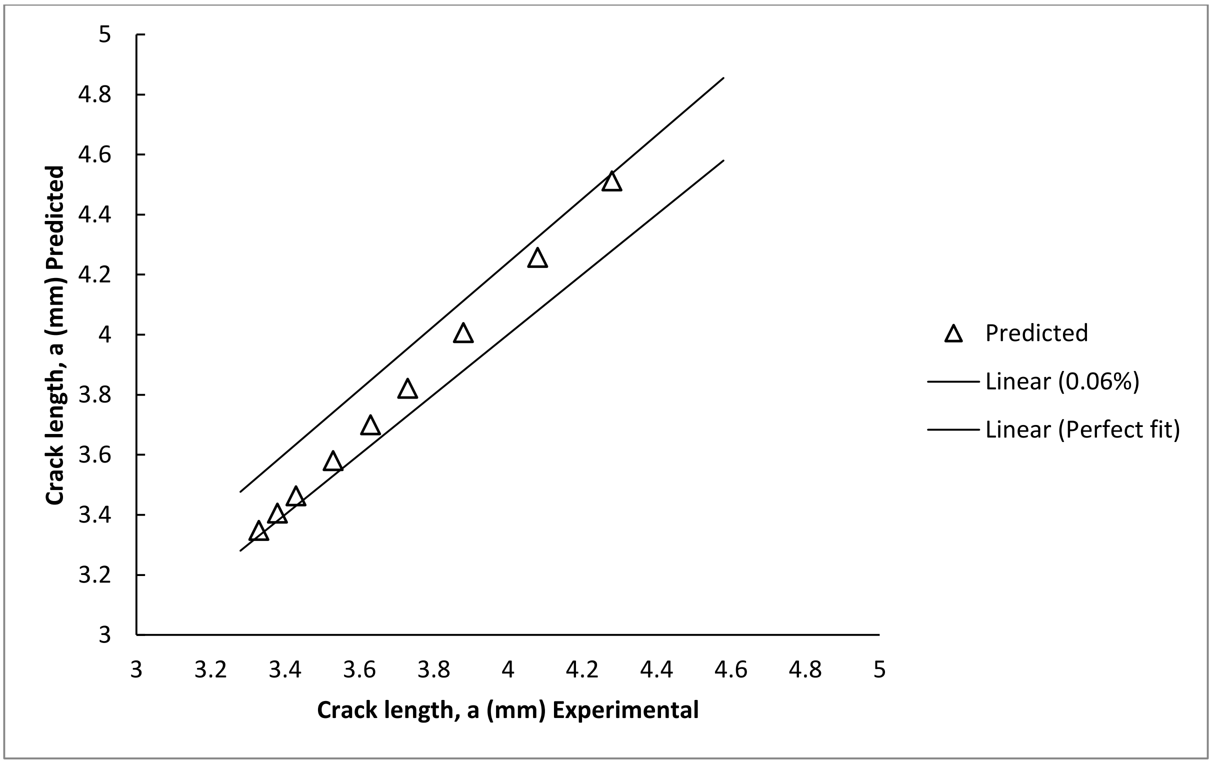

Fig. 7.3 Error band scatter for crack length (Exponential model)

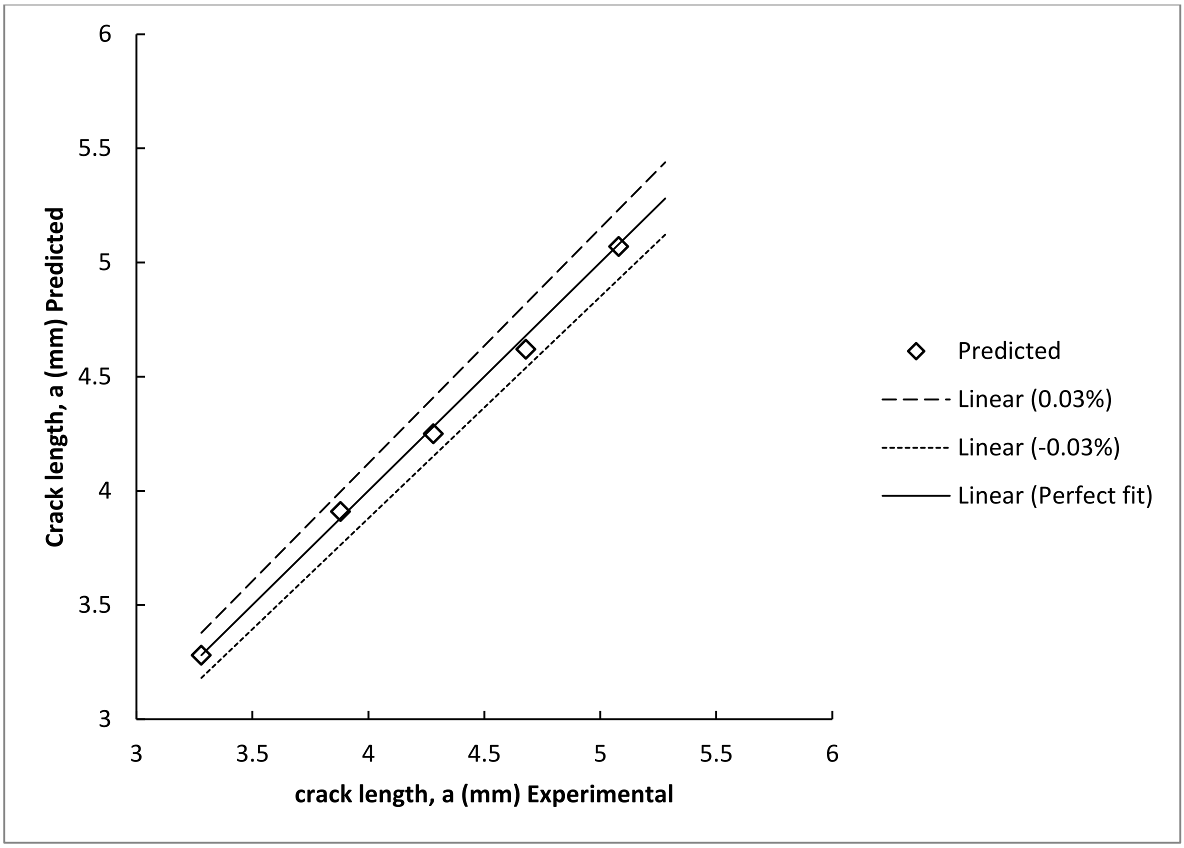

Fig. 7.4 Error band scatter for crack length (gamma model)

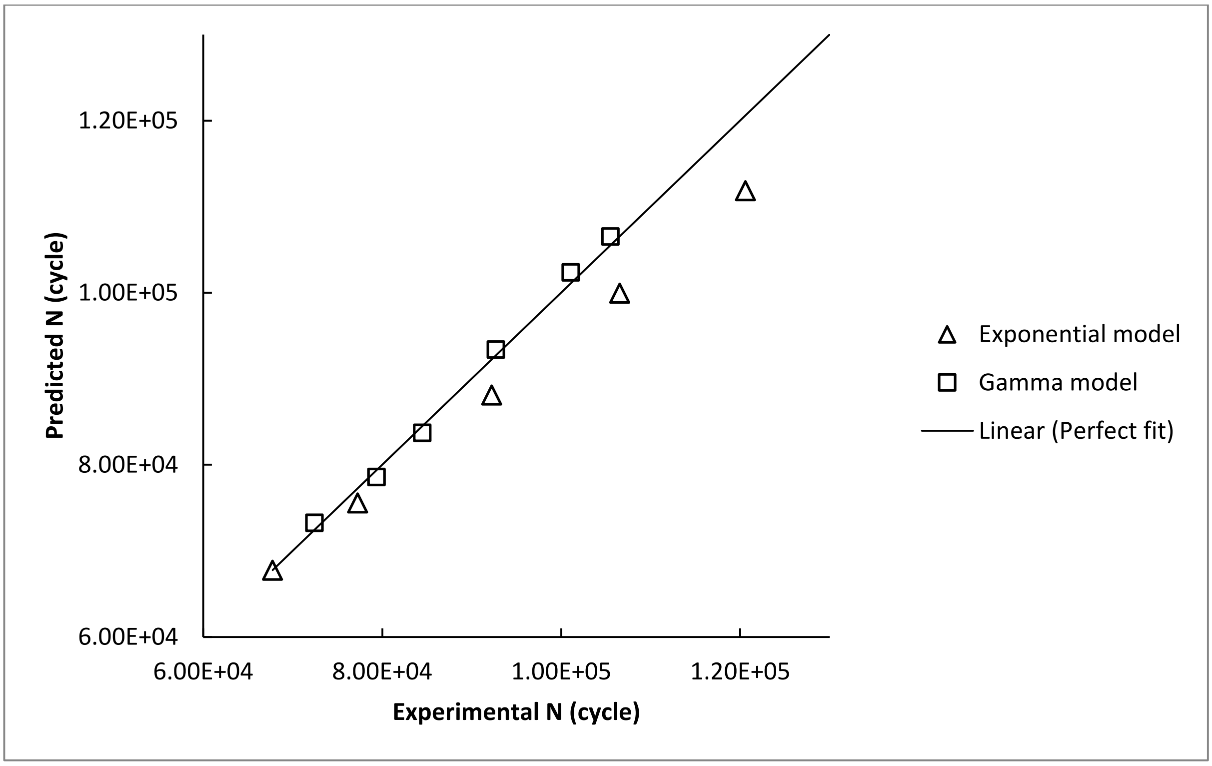

Fig. 7.5 Comparison of error band scatters of exponential model and gamma model (for number of cycle)

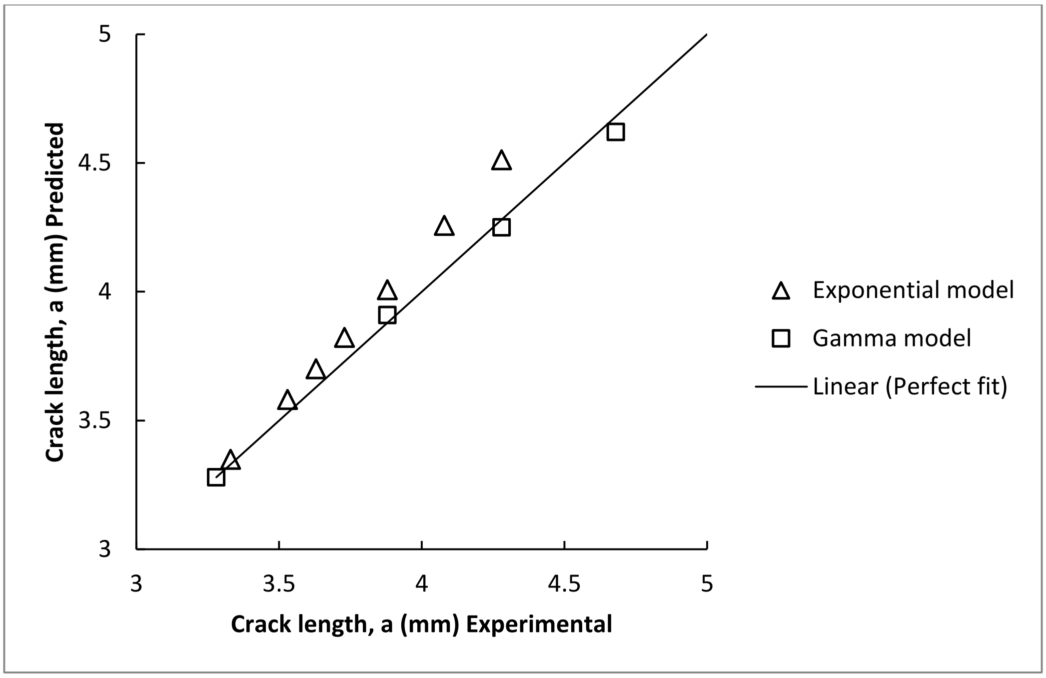

Fig. 7.6 Comparison of error band scatters of exponential model and gamma model (for crack length)

The fatigue crack growth rates (d*a*/d*N*) obtained from fitted experimental data. The Fig 7.7 & 7.8 shows d*a*/d*N* vs. del. *K* plot for part-through cracked pipe specimen.

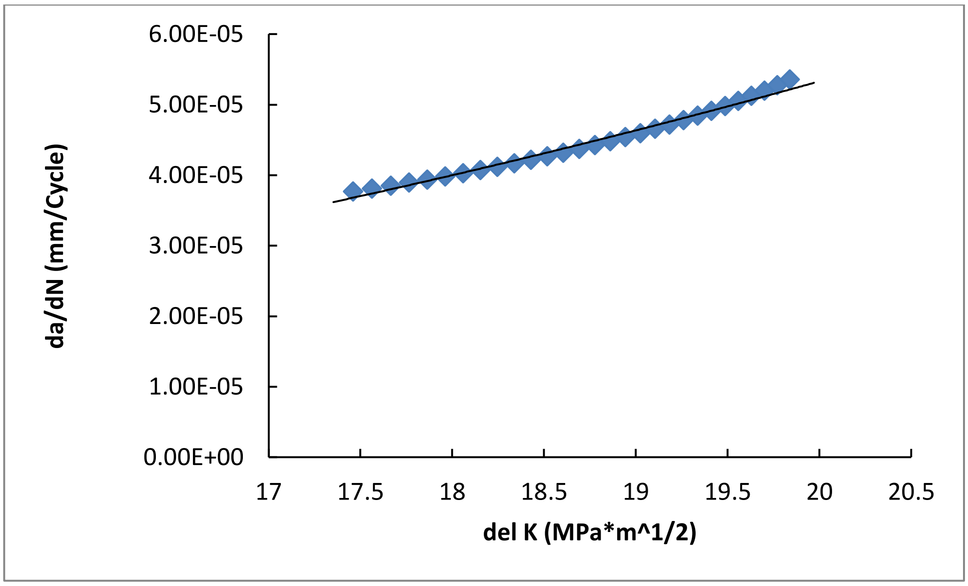

Fig. 7.7 Fatigue crack growth rate d*a*/d*N* vs. Δ*K* curve of TP316L stainless steel

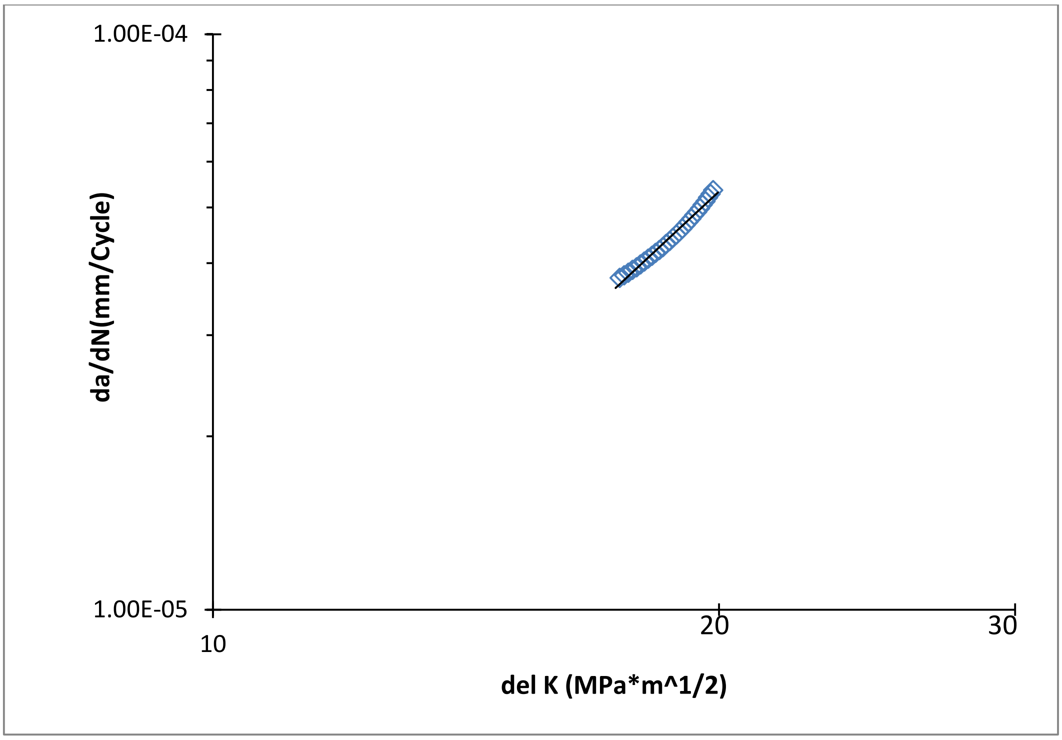

Fig. 7.8 Fatigue crack growth rate d*a*/d*N* vs. Δ*K* (log scale) curve of TP316L stainless steel

It is found that that the d*a*/d*N* vs. Δ*K* follows the general Paris Equation which is described as:

$$\frac{da}{dN} = 1.6 \times 10^{-8}(\Delta K^{2.72}) \tag{7.1}$$

The value of Paris law constants for part-through cracked pipe is given as:

$C = 1.6 \times 10^{-8}$

And,

n = 2.727

CHAPTER 8

CONCLUSIONS AND FUTURE WORK

CONCLUSIONS

1. Exponential model of the form $a_j = a_i e^{m_{ij}}(N_j - N_i)$ can be used to determine the fatigue crack propagation in part-through cracked pipes without going through numerical integration.
2. Gamma model of the form $\frac{ma_1}{w} = \int_0^N N^{\left(\frac{ma_0}{w}-1\right)} e^{-N} dN$ has been developed and effectively used to determine the fatigue crack propagation in part-through cracked pipes.
3. Specific growth rate (*m*) is an important parameter. In order to predict crack growth behaviour, it is necessary to define and correlate this parameter correctly with the physical variables that govern crack propagation.
4. Using the above models it is possible to predict the crack extension corresponding to a given number of cycles or to predict the number of cycles required for a given crack extension.
5. The prediction (number of cycles / crack length) made by exponential model is slightly conservative while the prediction made using gamma model lies in the vicinity of perfect fit line.
6. Both the models are capable of predicting crack growth behaviour satisfactorily. However, percent deviation for exponential model is marginally more than that for gamma model.

SUGGESTED FUTURE WORK

1. The proposed exponential model and gamma model may be tried to predict fatigue crack propagation under variable amplitude loading condition and different *R*-ratios in part-through cracked pipes.
2. The proposed models may be attempted for other specimen geometries.

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1. Pawan kumar, Vaneshwar Sahu, P. K. Ray, B. B. Verma, Prediction of fatigue crack growth in part-through cracked pipes using exponential model, NMD ATM 2012

2. Pawan Kumar, B.B. Verma, Hemendra Kumar Patel, P.K. Ray, Compliance correlation and prediction of fatigue crack growth in circumferentially cracked straight pipes, NCPCM 2011(at NIT Rourkela).

3. Shashi kumar, J. R. Mohanty, Pawan Kumar, P. K. Ray, Evaluation of fatigue crack growth and residual life using exponential model. 22ndlMRSIAGM-2011, Bhopal (at AMPRI Bhopal).

Paper Communicated

1. Pawan Kumar, Vaneshwar Sahu, B. B. Verma, P. K. Ray, "*Prediction of through the thickness fatigue crack growth in part through cracked pipes using exponential model*" is communicated to Applied Materials Today.